Capacitance and RC Circuits

Introduction A *capacitor* represents the first "circuit element" that we will study in addition to *resistors* and sources of *emf*. It is usually said that capacitors store charge. However, that requires some qualification since, under most circumstances, the net (total) charge on a "charged" capacitor is zero. Consequently, there are some subtleties that make it worthwhile to study capacitors more carefully. In fact, we will first study what determines *capacitance* and how to measure it. Next, we will spend some time learning about *series* and *parallel* arrangements of capacitors. Finally, we will study *RC circuits*.

Theory As you know, a *capacitor* is composed of two isolated electrical conductors (usually conductors of electrons rather than ions) separated by an insulator. When a source of *emf* is attached to the conductors, a charge, Q, is transferred from one conductor to the other, leaving a $+Q$ charge on one conductor and a $-Q$ on the other conductor. These charges create an electric field such that a potential difference (voltage) is established between the conductors. If we let V_{ba} be the *magnitude* of the potential difference, the *capacitance*, C, is defined by

Text eq. [24-1] $$Q = CV_{\text{ba}} \tag{1}$$

Consequently, C is the constant of proportionality between Q and V_{ba}. This general definition can be applied to capacitors where the conductors are specific shapes. One of the most common is a *parallel-plate capacitor*. For example, it is shown in our textbook that for parallel plates of area A separated by a distance d where an insulator of *dielectric constant, K*, fills the space between the plates, the capacitance is

Text eq. [24-8] $$C = K\epsilon_0 \frac{A}{d} \tag{2}$$

I. Parallel-Plate Capacitor Our first experiment will be to construct a parallel plate capacitor and make measurements of C, A and d in order to calculate K. The parallel plates will be thick aluminum plates, one of which is shown in the next picture. Teflon or nylon will be the insulator between the plates.

1. Try to decide which insulator is teflon and which is nylon. The teflon is "slippery." However, if you are still not sure which is which, note some distinguishing characteristic of each material (color, mark, scratch, etc.) and carry out the experiment. If the experiment is done carefully, the dielectric constant (which is a characteristic of a material) will indicate which is which.

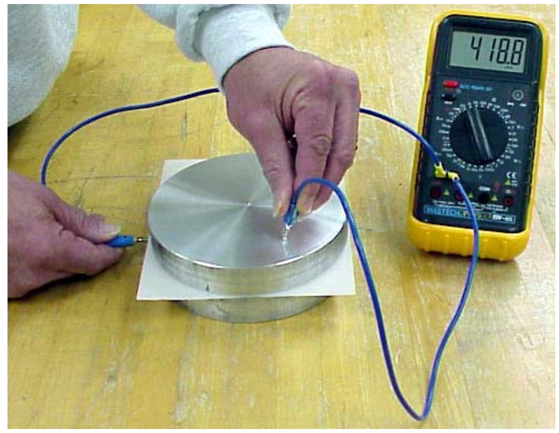
2. Use the digital calipers to measure the diameter of the plates. Record the average value in the space provided. Use the diameter to calculate the area and record the value in the space provided.

R1: *diameter* = _________ ± _________ m R2: A = _________ ± _________ m^2

3. Measure the thickness of the teflon and nylon insulators and record the values in the space provided.

R3: d_{Teflon} = __________ ± __________ m   R4: d_{nylon} = __________ ± __________ m

4. Sandwich one of the insulators between the two aluminum plates as shown in the picture.



5. The two leads with the copper strips at the end should already be connected to the Mastech multimeter. If not, insert the copper strips into the slots marked C_x on the front of the multimeter.

6. Turn on the meter and set the dial at 2000p on the F scale. The readings on the meter will be in picofarads (10^{-12}F).

We will measure the capacitance by touching one lead to an aluminum plate and the other lead to the other aluminum plate. A correct technique is shown in the picture. The following considerations are important.

- **Do not touch any metal portion of the leads or plates with your fingers. You are an electrical conductor and can contribute to the capacitance measured by the meter.**
- **Use the lead on the upper plate to press down on the plate with some force. The purpose is to flatten the insulator and eliminate air gaps to the extent possible.**
- **In the region between the meter and the capacitor, keep the leads as far apart as possible.** R5: **Why is it important to keep the leads as far apart as possible?**

7. Measure the capacitance and record the value in the space provided. (**Note**: According to the manual supplied with the multimeter, the accuracy of the capacitance measurements is 4%.)

R6: C_{Teflon} = __________ ± __________ pF   R7: C_{nylon} = __________ ± __________ pF

8. Carry out steps 4-7 for the other insulator and record the value in the space provided.

9. Use the data and eq. (2) to calculate the dielectric constant for each insulator and record the value in the space provided.

R8: $K_{\text{Teflon,measured}}$ = __________ ± __________

R9: $K_{\text{nylon,measured}}$ = __________ ± __________

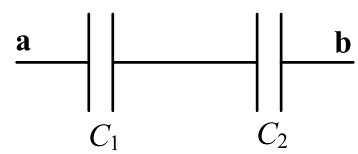
The accepted values for these constants are given in the previous textbook for this course, R. A.

Serway, *Principles of Physics*, 1998, Harcourt Brace College Publishers, Table 20.1 on page 584. Those values are

$K_{\text{teflon,accepted}} = 2.1$ $K_{\text{nylon,accepted}} = 3.4$

10. There are many types of teflon and nylon. Further, there is variability in the structure of each polymer. However, the agreement between the measured and accepted values should still be better than about 10%. R10: What is the percentage difference between the measured and accepted values? Show your calculation. Do the values allow you to distinguish between Teflon and nylon?

II. Capacitors in Series We will now study capacitors placed one after another i.e. in *series.* This is shown schematically at the right. Combinations of capacitors are often dealt with in terms of *equivalent capacitance*, C_{eq}. On pages 618 of our textbook, it is shown that the equivalent capacitance of series capacitors is given by



Text eq. [24-4]

$$\frac{1}{C_{\text{eq}}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \ldots \qquad (3)$$

We will now investigate the validity of this equation.

1. The capacitors that we will use in this part of the experiment are small, flat and blue and mounted on a Plexiglas® square. A picture is shown at the right. There are four different capacitors as may be obvious from their size. The clear, plastic mounting board of three of the capacitors is marked with a color. The fourth capacitor has no mark. Use the Mastech multimeter to measure the capacitance of each of the capacitors and record the values in the space provided. (Set the dial on the Mastech multimeter on either 2µ or 20µ on the F scale.)

R11: $C_{\text{black mark}}$ = ________ ± ________ µF R12: $C_{\text{blue mark}}$ = ________ ± ________ µF

R13: $C_{\text{yellow mark}}$ = ________ ± ________ µF R14: $C_{\text{no mark}}$ = ________ ± ________ µF

R15: How many times larger than C_{teflon} (measured in the previous part of this lab) is $C_{\text{no mark}}$ i.e. what is the ratio $C_{\text{no mark}}/C_{\text{teflon}}$?

R16: How large would the area of the teflon capacitor be if it had the same capacitance as the capacitor with no mark? Show your calculation. (The combination of the small size (dimensions) and large capacitance of the tiny, blue capacitors is an important recent achievement of science and technology.)

2. Construct a series combination of the two largest capacitors.

3. Use eq. (3) to predict the capacitance of the series combination between points **a** and **b**. Show your calculation and record the answer in the space provided.

R17: Calculation

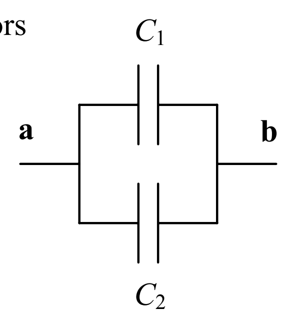
R18: $C_{\text{series,predicted}}$ = _________ ± _________ μF

4. Use the Mastech multimeter to measure the capacitance of the series combination between points **a** and **b**. Record the value in the space provided.

R19: $C_{\text{series,measured}}$ = _________ ± _________ μF

R20: Discuss whether your results confirm eq. (3).

III. Capacitors in Parallel We now consider capacitors that are placed side by side i.e. in *parallel*. This is shown schematically at the right. On pages 618 of the textbook, it is shown that the equivalent capacitance of parallel capacitors is given by



Text eq. [24-3] $$C_{eq} = C_1 + C_2 + C_3 \ldots \qquad (4)$$

We will now investigate the validity of this equation.

1. Construct a parallel combination of the two capacitors using the two capacitors used previously to form a series combination.

2. Use eq. (4) to predict the capacitance of the parallel combination between points **a** and **b**. Show your calculation and record the answer in the space provided.

R21: Calculation

R22: $C_{\text{parallel,predicted}} =$ ________ ± ________ μF

3. Use the Mastech multimeter to measure the capacitance of the parallel combination between points **a** and **b**. Record the value in the space provided.

R23: $C_{\text{parallel,measured}} =$ ________ ± ________ μF

R24: Do your results confirm eq. (4)? Discuss.

IV. RC Circuits The final part of the lab is to study a capacitor and resistor together in a circuit.

Theory The theory of the RC circuit is given in the textbook in section 26-4. One result of the theory is that when a resistor, capacitor and *emf*, $\mathcal{E}$, are connected in series, (This occurs if the switch is in position 1 in the next circuit.) the capacitor *charges* according to

Text eq. [26-5a] $$Q = C\mathcal{E}\left(1 - e^{-t/RC}\right) \tag{5}$$

The quantity $RC \equiv \tau$ is known as the *time constant*. Consequently, the voltage across the capacitor is

Text eq. [26-5b] $$V_C = \mathcal{E}\left(1 - e^{-t/RC}\right) \tag{6}$$

Finally, the associated *charging current* is

Text eq. [26-6] $$I(t) = \frac{\mathcal{E}}{R} e^{-t/RC} \tag{7}$$

If the battery is then removed from the circuit and the capacitor is connected across the resistor (This occurs if the switch is in position 2 in the next circuit.), the capacitor *discharges* according to

Text eq. [26-7] $$Q = Q_0 e^{-t/RC} \tag{8}$$

so that the voltage across the capacitor is

$$V_C = \frac{Q_0}{C} e^{-t/RC} \tag{9}$$

and the *current during discharge* is

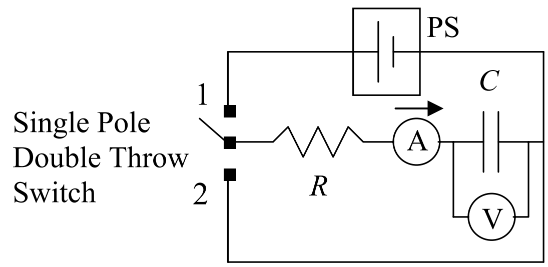
Text eq. [26-8] $$I(t) = I_0 e^{-t/RC} \tag{10}$$

We will measure the voltage across the capacitor and current through the resistor both while the capacitor is charging and while it is discharging.

Procedure

1. Construct the circuit shown in the next diagram. We will use the 25,000 μF (0.025 F) capacitor (shown at the upper right hand corner of the first page of this write-up) in the circuit for *C* and a 39 Ω resistor for *R*. Use the computer/DCA for both the voltage and current

measurements. Use the HP power supply for PS. Adjust the voltage of the power supply to about 4.5 V.



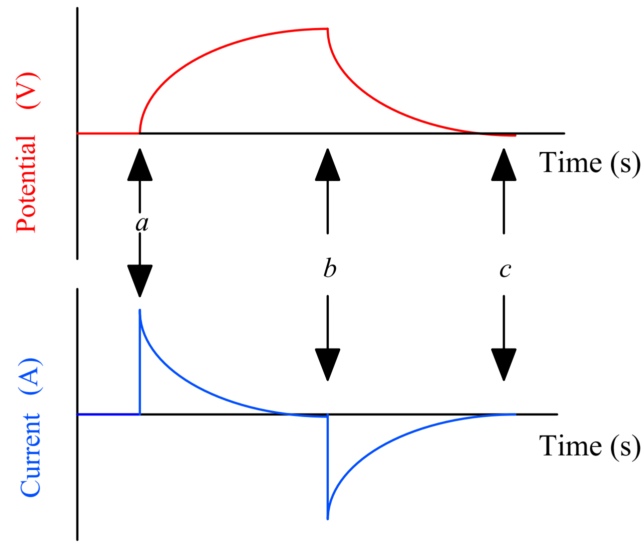
2. Note that the capacitance is larger and the capacitor is much bigger (has a larger physical size) than the capacitors studied previously.

R25: Explain why this is reasonable.

3. Start the program **LoggerPro**. When the program starts, open the folder **Capacitance** and open the file **RC Circ**.

4. Place the switch in position 2 for about 15 seconds and **Zero** all sensors.

5. Click on **Collect** then throw the switch to position 1. At about $t = 5$ s, throw the switch to position 2. The data should look something like the plot at the right. If you are not getting suitable data, trouble-shoot (try to find out what is wrong with) the circuit. If you are unable to solve the problem, consult your instructor.



Describe the results qualitatively by answering the following questions.

R26: What occurred in the circuit at time *a*?

R27: What current is being measured i.e. The current is through which circuit element?

R28: At (or just after) time *a*, what is the current? Why i.e. what should it be? (Please check the voltage from the power supply and show your calculation of the current at (or just after) time *a*.)

R29: What voltage is being measured i.e. The voltage is across what circuit element?

R30: What should the voltage be at time *a*? Explain.

R31: What is taking place in the circuit between points *a* and *b*?

R32: What occurred in the circuit at time *b*?

R33: What is taking place in the circuit between times *b* and *c*? Why is the current negative? Why is the voltage positive?

Data Analysis Next, we will carry out a quantitative analysis of the data. What we do is use the computer to "best-fit" various portions of the data. We begin with voltage.

1. Click the mouse anywhere on the voltage plot to be sure that voltage is the "active" plot.

2. Click on **Analyze** in the menu at the top of the computer screen and select **Automatic Curve Fit**. The voltage plot should appear in a box on the screen. If not, step 1 was a failure.

3. Click and drag the mouse across the screen between times *a* and *b*. Vertical lines should appear on the screen at times *a* and *b*.

4. In the equations box highlight (click on) the equation

A*(1-exp(-C*x))+B (11a)

This is labeled as Inverse Exponent in the list of equations. Note the difference in notation. In the notation of our textbook and the previous equations of this write-up, eq. (11a) is

$$\mathrm{A}\left(1-e^{-\mathrm{C}\mathrm{x}}\right)+\mathrm{B} \tag{11b}$$

This equation appears to be different than eq. (6). However, the differences are because, for our data, the voltage begins to increase at x = *a* (when the switch is first closed). However, eq. (6) describes voltage that begins to increase at *t* = 0. Consequently, x = *t* + *a* where *t* is the time in eq. (6). However, we will not bother with most the details of the relationship between eq. (11b) and eq. (6). The only quantity that we are interested in is C.

Beware: The C that appears in eqs. (11a) and (11b) is not the same as capacitance, *C*. For consistency of notation (with both the textbook and the computer program), we will only distinguish between C and *C* via italics for capacitance.

Since C multiplies the x in eqs. (11a) and (11b) it also multiplies the *t* in eq. (6). Consequently,

$$\mathrm{C} = \frac{1}{RC} = \frac{1}{\tau} \tag{12}$$

5. Click on **Try Fit**. The best-fit equation (equation containing numerical values of A, B and C) should appear on the screen. Also, the best-fit line should appear on the graph.

If the fit is not good (If the best-fit line and data are not very close to one another), it is probably because the portion of the data that was selected is not correct. Repeat steps 3-5 until the results are good.

6. The most important result is the value of C. Use the value of C and eq. (12) to calculate the measured value of the time constant. Record the values of C and τ in the space provided.

R34: $\mathrm{C}_{\text{increasing V}}$ = _________ ± _________ s^{-1}

R35: $\tau_{\text{measured, increasing V}}$ = _________ ± _________ s

7. Click and drag the mouse between the times *b* and *c*.

8. In the equations box highlight (click on) the equation

$$\text{A*exp(-C*x)+B} \tag{13a}$$

This is labeled as Natural Exponent in the list of equations. In the notation of our textbook equation (13a) is

$$\mathrm{A}e^{-\mathrm{Cx}} + \mathrm{B} \tag{13b}$$

R36: What is the relationship between x in eqs. (13a) and (13b) and *t* in eq. (9)? Why? (Because C once again multiplies *t*, eq. (12) again gives the relationship between C and τ.)

9. Click on **Try Fit**. Again, the best-fit equation and best-fit curve should appear on the screen. R37: Depending on the instructions from your instructor, either **Print** the results or **Copy** and **Paste** the results into an **Excel** spreadsheet so that the upper left hand corner is in cell A1. If you are using an Excel spreadsheet, scale the graphs so that they do not extend beyond column J.

10. Use the value of C and eq. (12) to calculate the measured value of the time constant. Record the values of C and τ in the space provided.

R38: $\mathrm{C}_{\text{decreasing V}}$ = _________ ± _________ s^{-1}

R39: $\tau_{\text{measured, decreasing V}}$ = _________ ± _________ s

11. Finally, use the given values of R and C (39 Ω and 0.025 F) to predict the time constant. Record the value in the space provided.

R40: $\tau_{\text{predicted}}$ = _________ ± _________ s

R41: Compare $\tau_{\text{predicted}}$ with $\tau_{\text{measured, increasing V}}$ and $\tau_{\text{measured, decreasing V}}$.

12. Change the resistance from 39 Ω to 22 Ω. Zero the probes then obtain data while charging the capacitor. R42: Depending on the instructions from your instructor, either **Print** the results or **Copy** and **Paste** the results into an **Excel** spreadsheet so that the upper left hand corner is in cell K1.

R43: Describe the effect of decreasing the resistance from 39 Ω to 22 Ω?

Suggestions for Further Work Determine the maximum charge on the capacitor two ways.
R44: First, predict the value using

$$Q_{\text{max,predicted}} = C\mathcal{E}$$

R45: Calculate the charge by finding the area under the current vs. time plot. This should be the same since

$$Q_{\text{max,measured}} = \int I dt$$

R46: Compare the predicted and measured values of the charge.

End of Lab Checkout Please do *not* disconnect the leads from the Mastech multimeter. Before leaving the laboratory, please dismantle any other connections that you have made. Tidy the lab bench.